Stochastic Pre-Hurricane Restoration Planning for Electric Power Systems Infrastructure

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***Abstract*—Proactive preparedness to cope with emergencies, especially those of nature origins, significantly improves the resilience and minimizes the restoration cost of electric power systems. In this paper, a proactive resource allocation model for repair and restoration of potential damages to the power system infrastructure located on the path of an upcoming hurricane is proposed. The objective is to develop an efficient framework for system operators to minimize potential damages to power system components in a cost-effective manner. The problem is modeled as a stochastic integer program with complete recourse. The large-scale equivalence of the original model is solved by the Benders' Decomposition method to handle computation burden. The standard IEEE 118-bus system is employed to demonstrate the effectiveness of the proposed model and further discuss its merits.**

***Index Terms*—Hurricane planning, power system resiliency, resource allocation, stochastic program with recourse.**

Nomenclature

b	Index for buses.
C_b	Hourly crew cost per person for bus b repair.
C_l	Hourly crew cost per person for line l repair.
C_{it}^{g}	Generation cost of unit i at time t.
C_{it}^{sd}	Shutdown cost of unit i at time t.
C_{it}^{su}	Startup cost of unit i at time t.
D_{bt}	Load demand at bus b at time t.
i	Index for generation units.
I_{it}	Commitment state of generating unit i at time t; 1 if committed, otherwise 0.
l	Index for transmission lines.
LI_{bt}	Load interruption at bus b at time t.
M	Large positive constant.
N_b	Set of components connected to bus b.
$p(s)$	Probability of scenario s.
P_{it}	Real power generation of unit i at time t.
P_{it}^{max}	Maximum generation capacity of unit i .
P_{it}^{min}	Minimum power generation capacity of unit i.
PL_{lt}	Power flow of line l at time t.
R_t^{max}	Number of available repair crew at time t.
R_t	Size of repair crew allocated to a component at time t.
s	Index for scenario.
SD_{it}	Shutdown cost of unit i at time t.
SU_{it}	Startup cost of unit i at time t.
t	Index for time.
T	Random time to repair for each component.
$VOLL_{bt}$	Value of lost load at bus b at time t.
u_{bt}	Repair state variable of bus b at time t; 1 if on repair, otherwise 0.
v_{lt}	Repair state variable of line l at time t; 1 if on repair, otherwise 0.
X_t^+	Secondary resource state penalty variable.
w_{lt}	Outage state of line l at time t; 0 if damaged, otherwise 1.
y_{it}	Outage state of unit i at time t; 0 if damaged, otherwise 1.
z_{bt}	Outage state of bus b at time t; 0 if damaged, otherwise 1.
α_{ib}	Element of unit i and bus b in generation-bus incidence matrix.
β_{lb}	Element of line l and bus b in line-bus incidence matrix.
γ	Probability of damage of a component.
δ_{bts}	Bus voltage angle.
ϑ_i	Random initial state of unit i after hurricane strikes; 0 if damaged, otherwise 1.
ξ	A multivariate random variable.
ϕ_b	Random initial state of bus b after hurricane strikes; 0 if damaged, otherwise 1.
ψ_l	Random initial state of line l after hurricane strikes; 0 if damaged, otherwise 1.

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I. Introduction

Natural disasters and extreme weather events, in particular hurricanes, result in significant economic, social, and physical disruptions and cause considerable inconvenience for residents living in disaster areas due to loss of electricity, water, and communication [1]. Therefore, it calls for a comprehensive study of this issue from different perspectives to find efficient ways of improving the resilience of these critical lifeline systems.

Various studies have been proposed in the literature in the context of emergency planning for power systems. In [2], the research problems and models for substations and/or distribution feeders planning under normal and emergency conditions were reviewed and discussed. A case study on hurricane planning and rebuilding the electrical infrastructure along the Gulf Coast for hurricane Katrina was presented in [3]. A risk assessment method for infrastructure technology planning to improve the power supply resiliency to natural disasters was proposed in [4]. Reduced cost as well as power supply availability were considered as two fundamental decision factors in their hurricane planning approach. In [5] a stochastic integer program was proposed to find the optimal schedule for inspection, damage evaluation, and repair of post-earthquake damaged electric power system. The aim was to minimize the average time that each customer is without power. A comprehensive survey of models and algorithms for emergency response logistics in electric distribution systems, including reliability planning with fault considerations and contingency planning models, were presented in [6], [7].

In the context of physical behavior analysis of power system infrastructure in hurricane disaster, [8] analyzed the resilience of power distribution systems based on the power distribution infrastructure and its interaction with the biophysical environment, and the way that restoration processes are prioritized. It was concluded that even though the infrastructure does not have any significant effect on outage duration, the interaction between infrastructure and the biophysical environment significantly affects on outage duration. In another study, a data mining approach to evaluate the impact of soil and topographic variables on accuracy of the power outage prediction models in hurricane events was proposed [9]. The results indicate that certain land cover variables can be approximated for the power system and be incorporated in the model when detailed information about the power system is not available. In [10] a method for characterization of the behavior of networked infrastructure, including power delivery systems in natural hazard events such as hurricanes, was presented. The model also included resilience and interdependency measures. The model can be utilized to develop design strategies for improved power infrastructure resiliency in natural disasters. [11] proposed a probabilistic framework for vulnerability analysis of distribution poles subject to hurricane hazards considering the impact of changing climate. The results indicate that changing climate and the age of the poles significantly increases the failure rate of distribution poles.

Outage prediction is an important means in order to have an efficient response to the hurricane. In this context, [12] introduced a method for estimating the restoration time of electric power systems after hurricanes and ice storms. Using a large dataset of six hurricanes and eight ice storms, the accelerated failure time models were developed to forecast the duration of each probable outage. In [13] the negative binomial regression models for prediction of outages due to hurricane were developed. The number of transformers in the area, maximum gust wind speed, the power company affected, and a hurricane effect turned out to be the most explanatory variables. Diagnostic statistics such as pseudo R-squared values were used for model selection purposes. Their adopted zip code based model can be used for prediction of the likely outage rates prior to the hurricane events. In another work, [14] used regression analysis and data mining to develop models to estimate the number of utility poles that will be damaged based on damage data from past storms. Results indicate that the hurricane-prone damages to the poles can be predicted in an accurate manner, given that past damage data are available and adequate. However, the availability of past data can be a challenging issue, which limits the models efficiency in practice.

In the context of resource allocation for restoration of power systems, [15] presented three mathematical goal programming models in order to locate the repair units and restore the transmission and distribution lines in an efficient manner. The first model finds the optimal repair-unit dispatch tactical plan with a forecast of adverse weather conditions. The second model derives the optimal repair-unit location for a short-term strategic plan under normal weather conditions. The third model finds the optimal number of repair units for a long-term strategic plan. In another work, a mixed-integer programming model and a general column-generation approach for inventory decision making of power system components throughout a populated area for maximization of the amount of power served after disaster restoration was proposed [16]. In [17] the service restoration considering the restrictions on emergency-response logistics was studied with the objective of minimizing the customers interruption cost. The reconfiguration and the resources dispatching issues were considered in a systematic way in order to derive the optimal time sequence for every step of the restoration plan. In [18] a decision-making model to manage the required resources for economic power restoration operation was proposed. The optimal number of depots, the optimal location of depots, and the optimal number of repair crews were determined by their model in order to minimize the transportation cost associated with restoration operation. In [19] a decision support tool for improvement of information used by electric utilities for managing restoration of power distribution components damaged due to large-scale storms was described. The circuit layout, the placement of protective and switching devices, and the location of customers were taken into account to allocate the crew resources in order to manage the storm outage in a cost-effective manner.

Although variety of problems for power system planning in hurricane-prone events have been addressed in the literature, to the best of our knowledge a few provide a comprehensive and generic approach for resource allocation. In this paper, an efficient decision making tool is developed for restoration planning of power systems to minimize the expected customer load interruption cost, restoration operation cost, and electricity generation cost. The physics and the economy of the power system, specifically the unit commitment problem are incorporated into our model,which results in higher practicality of the proposed model for utility companies.

The rest of this paper is organized as follows: Section II describes the proposed model and Section III presents the problem formulation and methodology. Section IV illustrates the numerical results on the IEEE 118-bus test system. Finally, Section V provides the concluding remarks.

II. Model Description

Consider a power system whose some of its components, including generation units, transmission lines, and substations along with downstream distribution lines, are located on the path of an upcoming hurricane. The objective is to proactively allocate and mobilize available resources to enable quick response capability of system operators to repair and restore potential damages, in a way that minimizes the expected incurred costs. Furthermore, estimation of additional resources which need to be outsourced in order to cope with the aftermath of the hurricane is another critical issue for system operators. The expected incurred cost composed of the customers load interruption cost, electrcity generation cost, and the system repair and restoration cost.

In this paper, the damage state of components are represented by Bernoulli random variables and are considered to have two states: *damaged* and *functional*. If after the upcoming hurricane the component is still *functional*, the value of 1 will be assigned; and if the component is *damaged*, this value will be 0. Weather-related failure rate and probability of component damages follow models in [1].

The time to repair for each potentially damaged component is considered to be stochastic and is modeled by a random variable that may take various probability distributions. In this paper, without loss of generality, it is assumed that the time to repair random variables to be defined by the Weibull density function as follows

$$f_{T_k}(t) = \begin{cases} \frac{\rho}{\lambda_k}\left(\frac{t}{\lambda_k}\right)^{\rho-1} e^{-(t/\lambda_k)^\rho} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where ρ is the shape parameter, λ_k is the scale parameter, and $k \in \{b, i, l\}$. Any other probability distribution can be used without loss of generality. Considering the stochastic nature of the pre-hurricane problem, resources need to be allocated in a way that minimizes the expected cost of system restoration. However, at the time of resource allocation the damage state and the time to repair of components are not yet realized. Moreover, there are set of decision variables that need to be determined in the later stage once the outcomes of the hurricane are known. Therefore, the problem structure would be a two-stage stochastic problem with recourse. The objective will be to allocate resources in a way that minimizes first-stage resource allocation costs as well as expected recourse costs. In this class of problems, for each possible stochastic circumstance, a set of recourse (second-stage) activities can be performed to compensate the violation of the prevailing constraints [20].

III. Problem Formulation and Methodology

The problem is formulated as a two-stage stochastic linear program with recourse. The general formulation of a two-stage stochastic linear program with recourse is as follows

$$\begin{aligned} \min \quad & z = cx + Q(x) \\ \text{s.t.} \quad & Ax = b,\ x \in X, \end{aligned} \quad (2)$$

where $\mathbf{c}$ is the cost vector, $\mathbf{b}$ is the right hand side vector, $x = [{u_{bt}}^T, {v_{lt}}^T]^T$ is the first stage decision variable, $\mathbf{A}$ is the coefficient matrix of the first stage variable (which in our model is the cost coefficient of the resource constraint), and function $Q(x)$ is the second stage value function (expected recourse cost function) defined as

$$Q(x) = \mathbb{E}_\xi[Q(x, \xi(\omega))] \quad (3)$$

with

$$Q(x, \xi(\omega)) = \min_y \{q(\omega)y | Wy = h(\omega) - T(\omega)x,\ y \in Y\} \quad (4)$$

where $y = [{LI_{bts}}^T, {I_{its}}^T, {P_{its}}^T, {SU_{its}}^T, {SD_{its}}^T, {X^+_{bts}}^T, {X_{lts}^+}^T]^T$ is the second stage variable of the proposed model, $q(\omega)$ is the recourse penalty coefficient, $\mathbf{W}$ is the recourse matrix (which is the coefficient matrix of the second stage variables in the recourse problem's constraints) $\mathbf{T}$ is the technology matrix (which is the coefficient matrix of the first stage variables in the recourse problem's constraints), and ξ is a random N-vector in (Ω, A, S) probability space (which in our model is the multivariate random variable of damage state and time to repair for all components under hurricane damage risk) [21].

A. Objective function

The objective of the pre-hurricane model is to minimize the cost of the first stage resource allocation decisions, and the expected cost of second stage system configuration as follows:

$$\begin{aligned} \min_{u,v} & \sum_t \sum_b C_{bt} \cdot R_{bt} \cdot u_{bt} + \sum_t \sum_l C_{lt} \cdot R_{lt} \cdot v_{lt} \\ & + \mathbb{E}_S \Bigg[\min_{LI,I,P,SU,SD,X_l^+,X_b^+} \sum_t \sum_b VOLL_{bt} \cdot LI_{bts} \\ & \quad + \sum_t \sum_i \left(C^g_{it} \cdot I_{its} \cdot P_{its} + SU_{its} + SD_{its} \right) \\ & \quad + \sum_l \sum_t q^+_{lt} \cdot X^+_{lts} + \sum_b \sum_t q^+_{bt} \cdot X^+_{bts} \Bigg], \end{aligned} \quad (5)$$

The first term represents the cost of resources primarily allocated to substations (and their downstream distribution lines), and the second term is the cost of resources primarily allocated to transmission lines. The expected second-stage (recourse) function includes the load interruption cost over the restoration planning horizon, and the total generation cost including fuel costs, startup costs, and shutdown costs in scenario s. It further includes the cost of secondary resources that are allocated to transmission lines and substations under scenario s. These secondary resources fill the shortage of actual restoration resources that have not been allocated in the first stage decisions, but are required to accomplish the restoration operations under scenario s. Mathematically, these secondary resources eliminate infeasibility of the second stage problem under any decision which is made in the first stage.

B. Constraints

1) Resource constraint: The first stage problem is constrained by restriction on primary resources (6) which represents the maximum amount of resources that can be allocated to the entire system in each time period.

$$\sum_{l} R_{lt} \cdot v_{lt} + \sum_{b} R_{bt} \cdot u_{bt} \leq R_t^{max}, \ \forall t. \quad (6)$$

2) Damage state of generation units: The initial damage state of each generating is represented by the outcome of random variable $\vartheta_i \sim Bernoulli(\gamma_i)$. The damage state of generation units over the restoration horizon is modeled as follows:

$$t - M \cdot y_{its} \leq (1 - \vartheta_i^s) \cdot T_i^s, \ \forall i, \forall t, \forall s, \quad (7)$$

$$y_{its} \leq \vartheta_i^s, \ \forall i, \forall t = 0, ..., T_i, \forall s, \quad (8)$$

where T_i^s is the outcome of random variable $T_i \sim Weibull(\lambda_i)$ in scenario s. If at the beginning of the restoration horizon of scenario s, the generation unit i is in *damaged* state, then the outcome of random variable ϑ_i denoted by ϑ_i^s is 0; hence it needs to be restored. The state of a damaged generating unit does not change, unless the required restoration operation is performed. On the other hand, if the generation unit is *functional*, then the random variable ϑ_i is 1 which indicates that it can be immediately committed for generation. It is assumed that if a generation unit is in the *functional* initial state, its state will remain the same up to the end of the restoration horizon for that particular scenario.

3) Damage state of substations: The initial damage state of each substation b under scenario s, z_{b0s} is represented with the outcome of random variable $\phi_b \sim Bernoulli(\gamma_b)$. The damage state of each substation over the restoration horizon under scenario s is modeled as follows:

$$z_{b0s} = \phi_b^s, \ \forall b, \forall s, \quad (9)$$

$$0 \leq z_{b(t+1)s} - \left(\sum_{k=1}^{t} \left(u_{bk} + X_{bks}^{+} \right) - T_b^s + 0.5 \right) / M \leq 1 \quad \forall b, \forall t, \forall s. \quad (10)$$

$$\sum_{k=t}^{t+T_b^s-1} \left(u_{bk} + X_{bks}^{+} \right) \geq T_b^s \Big(u_{bt} + X_{bts}^{+} - u_{b(t-1)} - X_{b(t-1)s}^{+} \Big), \ \forall b, \forall t, \forall s, \quad (11)$$

where T_b^s is the outcome of random variable $T_b \sim Weibull(\lambda_b)$, and u_{bt} is the first stage decision variable on primary resource allocation to bus b at time t; and

$$u_{bt} + X_{bts}^{+} \leq 1, \ \forall b, \forall t, \forall s. \quad (12)$$

Constraint (9) models the damage state of a substation during restoration horizon, while (10) guarantees that enough resources are allocated to the component for repair purposes. It is assumed that if a substation under a scenario s is initially in *functional* state ($z_{b0s} = 1$), it will remain in the same state up to the end of the restoration horizon.

4) Damage state of transmission lines: In the same way as generation units and substations, the initial damage state of transmission lines is represented by the outcome of a random variable $\psi_l \sim Bernoulli(\gamma_l)$. The damages state and repair duration of transmission lines in each scenario are modeled as follows:

$$w_{l0s} = \psi_{ls}, \ \forall l, \forall s, \quad (13)$$

$$0 \leq w_{l(t+1)s} - \left(\sum_{k=1}^{t} \left(v_{lk} + X_{lks}^{+} \right) - T_{ls} + 0.5 \right) / M \leq 1 \quad \forall l, \forall t, \forall s, \quad (14)$$

$$\sum_{k=t}^{t+T_{ls}-1} \left(v_{lk} + X_{lks}^{+} \right) \geq T_{ls} \Big(v_{lt} + X_{lt}^{+} - v_{l(t-1)} - X_{l(t-1)s}^{+} \Big), \ \forall l, \forall t, \forall s, \quad (15)$$

where T_l^s is the outcome of random variable $T_l \sim Weibull(\lambda_{ls})$, and

$$v_{lt} + X_{lts}^{+} \leq 1, \ \forall l, \forall t, \forall s. \quad (16)$$

If a transmission line is damaged, its initial state variable (w_{l0s}) becomes 0, and remains unchanged until required resources are allocated and repaired the damaged component. On the other hand, if it does not undergo any damage, w_{l0s} takes the value of 1. It is assumed that the *functional* state of a component remains unchanged during the restoration horizon.

5) Load balance constraint: The bus load balance constraint for each scenario is represented as follows:

$$\sum_{i \in N_b} P_{its} + \sum_{l \in N_b} PL_{lts} + LI_{bts} = D_{bt}, \ \forall b, \forall t, \forall s. \quad (17)$$

This constraint ensures that the injected power to a bus from connected transmission lines and generation units must supply the bus load in each scenario; however, if the injected power is not sufficient, the load supply will be interrupted equal to the load interruption variable (LI_{bts}).

6) Power generation constraints: The real power generation of unit i is constrained with its damage state, the commitment state, and its minimum and maximum generation capacity as follows

$$P_i^{min} \cdot y_{its} \cdot I_{its} \leq P_{its} \leq P_i^{max} \cdot y_{its} \cdot I_{its}, \ \forall i, \forall t, \forall s. \quad (18)$$

The coupling constraint of unit commitment and damage state holds for each scenario as

$$I_{its} \leq y_{its}, \ \forall i, \forall t, \forall s. \quad (19)$$

The damage state of substations connected to each generating unit also constrains the real power generation as

$$-M \sum_b \alpha_{ib} \cdot z_{bts} \leq P_{its} \leq M \sum_b \alpha_{ib} \cdot z_{bts}, \ \forall i, \forall t, \forall s, \quad (20)$$

Thus, if the substation connected to a generation unit is damaged, the generating unit becomes offline.

7) Power flow constraints: Transmission network power flow is modeled as follows

$$-PL_l^{max} \cdot w_{lts} \leq PL_{lts} \leq PL_l^{max} \cdot w_{lts}, \ \forall l, \forall t, \forall s, \quad (21)$$

$$-M \sum_b \beta_{lb}^{from} \cdot z_{bts} \leq PL_{lts} \leq M \sum_b \beta_{lb}^{from} \cdot z_{bts}, \quad \forall l, \forall t, \forall s, \quad (22)$$

$$-M \sum_b |\beta_{lb}^{to}| \cdot z_{bts} \leq PL_{lts} \leq M \sum_b |\beta_{lb}^{to}| \cdot z_{bts}, \forall l, \forall t, \forall s, \quad (23)$$

$$\begin{aligned} &-M(1-w_{lts}) - M(1-\sum_b |\beta_{lb}| \cdot z_{bts}) \\ &\quad \leq PL_{lts} - \frac{\sum_b \beta_{lb} \cdot \delta_{bts}}{x_l} \leq \\ &M(1-w_{lts}) - M(1-\sum_b |\beta_{lb}| \cdot z_{bts}), \ \forall l, \forall b, \forall t, \forall s, \quad (24) \end{aligned}$$

If line l at time t is in the *functional* state, the power can be flowed in either of the two directions, but not more than the maximum power flow capacity of the line (21). However, if the line is damaged, the line flow will be equal to 0. In addition, as long as any of the substations connected to each particular transmission line is in the *damaged* state, the associated line flow would be set to 0 (22)-(23). The transmission line flow is related to bus voltage angles as in (24).

8) Unit commitment constraints: The unit commitment constraints for thermal generating units are important feature of the hurricane restoration planning model. The related MIP constraints, i.e., startup and shutdown costs, ramp-up and ramp-down, and minimum uptime and downtime constraints are imposed to the model in order to incorporate the impact of optimal unit commitment configuration of the system in restoration decision making [22].

9) Nonanticipativity: An important issue that needs to be considered in solving stochastic programs is that the decisions should not depend on the outcome of stochastic parameters, denoted as the nonanticipativity concept [20]. One way to enforce the nonanticipativity requirement is the Birge's method [23]. For instance, for LI_{bts} the nonanticipativity is modeled as follows:

$$\left(\sum_{\acute{s} \in S_s^t} p(\acute{s}) \right) \cdot LI_{bts} = \sum_{\acute{s} \in S_s^t} p(s) \cdot LI_{bt\acute{s}}, \ \forall b, \forall t, \forall s, \quad (25)$$

where S_s^t is the set of scenarios that are identical to scenario s at time t. A similar formulation structure for the rest of the second stage variables is considered in the model.

C. The proposed scheme

1) Scenario construction and reduction: Due to presence of continues random variables, i.e., the Weibull distribution for time to repair of each damaged component, the stochastic data process of the proposed models, ξ has an infinite support. To make the problem tractable, the stochastic data process ξ needs to be redistributed to provide a finite support with the reduced (optimal) number of scenarios. We use the Latin hypercube sampling [24] to replace ξ by a scenario tree approximation ξ_{tr} which has a finite, but large number of scenarios. The Latin hypercube sampling guarantees that the whole range of values for a random variable is sampled. For a sample size of N, the Latin hypercube sampling technique selects N different values from each of random variables by dividing the range of each random variable into N non-overlapping intervals. Then by shuffling and pairing these values constructs N scenarios, each with probability of $1/N$.

The next step is to reduce the number of scenarios into a computationally tractable size. Various reduction techniques are available to be applied for different applications. For the constructed probability measure of $\xi_{tr} = \sum_{k=1}^{N} \frac{1}{N} s_k$, it is required to determine an index set $K_*^{'} \subset \{1, ..., N\}$ of given cardinality $\#K_*^{'} = N - N^{'}$ and a probability measure $\tilde{\xi}_* = \sum_{k'=1, k' \notin K_*'}^{N} p_{k'}^* s_{k'}$ such that

$$\begin{aligned} \hat{\mu}_c(\xi_{tr}, \tilde{\xi}_*) = \inf \Bigg\{ &\hat{\mu}_c \Bigg(\xi_{tr}, \sum_{k=1, k \notin K_*'}^{N} p_{k'} s_{k'} \Bigg) : \\ &K_*^{'} \subset \{1, ..., N\}, \#K_*^{'} = N - N^{'}, \\ &\sum_{k' \notin K_*'} p_{k'} = 1, p_{k'} \geq 0, k^{'} \notin K^{'} \Bigg\} \quad (26) \end{aligned}$$

where *Kantorovich functional* $\hat{\mu}_c(\xi_{tr}, \tilde{\xi}_*)$ is an estimation of the probability distance $\zeta_c(\xi_{tr}, \tilde{\xi}_*)$. Problems (26) can be solved through variety of techniques, but due to accuracy of backward reduction algorithm, we solve it through this method. Readers are referred to [26] for the detailed explanation of the backward scenario reduction algorithm.

Algorithm 1 Benders' decomposition for two-stage problem with recourse

{initialization}
Lower bound (LB) $:= -\infty$, Upper bound (UB)$:= +\infty$
Solve initial IP master problem
$\min_{u,v}\{first\ stage\ problem\}$
$\bar{u} := u^*,\ \bar{v} := v^*$ {optimal values}

while UB $-$ LB $> \varepsilon$ **do**
for $s \in S$
{solve MIP second stage problem for each scenario s}
end for

UB $:= \min\{$UB, $\sum_t \sum_b C_{bt}.R_b^{min}.\bar{u}_{bt} + \sum_t \sum_l C_{lt}.R_l^{min}.\bar{v}_{lt} + p(s)[\sum_t \sum_b VOLL_{bt}.\bar{LI}_{bts} + \sum_t \sum_i (C_{it}^g.\bar{I}_{its}.\bar{P}_{its} + \bar{SU}_{its} + \bar{SD}_{its}) + \sum_l \sum_t q_{lt}^+.\bar{X}_{lts}^+ + \sum_b \sum_t q_{bt}^+.\bar{X}_{bts}^+]\}$

Add cut $\theta \geq \sum_s p(s)(-\pi_s.[T_s x + W_s y - h_s])$

{solve the master problem
$\min_{u,v,\theta} \sum_t \sum_b C_{bt}.R_b^{min}.u_{bt} + \sum_t \sum_l C_{lt}.R_l^{min}.v_{lt} + \theta$
s.t. cuts and original constraints}

$\bar{u} := u^*, \bar{v} := v^*, \bar{\theta} := \theta^*$ {optimal values}
LB $:= \sum_t \sum_b C_{bt}.R_b^{min}.\bar{u}_{bt} + \sum_t \sum_l C_{lt}.R_l^{min}.\bar{v}_{lt} + \bar{\theta}$
end while

2) Decomposition strategy: Due to the large scale of the deterministic equivalence of the proposed model, using Benders' decomposition method [27], the problem is decomposed into a master problem which includes the first stage problem, and $N - N^{'}$ (i.e. the number of reduced scenarios) subproblems. We use the Benders' decomposition algorithm for two-stage problem with complete recourse, as shown in Algorithm 1. The first stage problem is only involved with u_{bt} and v_{lt} variables and corresponding resource constraints, while the subproblem for each scenario s is composed of the rest of the recourse function with the rest of the constraints of the model. The constraints with θ represent the optimality cuts; and π_s represents the dual variable of the scenario s. Since the problem has a complete recourse, thus the subproblems are always feasible. Therefore, there is no need for feasibility cut in the decomposition algorithm. This iterative algorithm is continued until desired optimality gap is obtained.

IV. Numerical Analysis

The IEEE 118-bus system is considered to study the proposed model. The component of the system located on the path of the upcoming hurricane along with the associated probability of damage, as well as the scale parameter of the Weibully distributed time to repair are given in the first three columns of Tables I-III. The shape parameter of the Weibull distribution for all components are assumed to be equal to 1. The value of lost load is considered to be \$3706/MWh for industrial loads, \$6979/MWh for commercial loads, and \$110/MWh for residential areas [28]. The load on bus B62 is industrial, while loads on buses B88, B92, and B93 are commercial. The rest of the loads are considered as residential. The repair crew is considered to be the only limited resource that is allocated to repair damaged components. It is assumed that each damaged substation requires 10 repair crews/hour, while each damaged transmission line requires 15 repair crews/hour. The wages for repair crews of substations are as assumed follows: \$60/hour at shift 1 (8:00 A.M.–4:00 P.M.), \$70/hour at shift 2 (4:00 P.M.–12:00 A.M.), and \$80/hour at shift 3 (12:00 A.M.–8:00 A.M.). An incremental rate of \$5/hours is added to the wage of transmission lines' repair crew. The generation cost is considered to be \$35.09/MWh [29]. The shutdown cost for each generating unit is assumed to be \$250. The startup cost is assumed to be \$150 within the first hour after last shutdown. For each additional hour (up to eight hours), an incremental cost of \$25 is added to the startup cost. Three cases are considered for analysis as follows:

Case I (full restoration): The primary resources are considered as unconstrained. The restoration of all components are enforced. The aim is to find the optimal value for the maximum amount of resources required for full restoration.

Case II (partial restoration): The primary resources are constrained to obtained value in Case I. The restoration is not enforced for all damaged components. The aim is to analyze the economic dynamics of the restoration, regardless of system-level reliability.

Case III (expected value problem): The expected value problem of the proposed model is solved. The aim is to find the value of stochastic solution (VSS) for the problem.

All three cases are analyzed in a 120-hour restoration planning horizon. For Cases I and II, 3000 independent scenarios are generated using the Latin hypercube sampling method [25]. With the use of the backward reduction algorithm [26], the number of scenarios are reduced to 10, with associated probabilities of 0.336, 0.011, 0.017, 0.065, 0.029, 0.308, 0.115, 0.012, 0.057, and 0.05. All three cases are solved using the Benders' decomposition method. First, the proposed model is solved for Case I to find $R^{max}(t)$, i.e., the maximum resource level required to restore the entire system. After obtaining $R^{max}(t)$, this value is imposed as a constraint in Case II in order to study system behavior, when the system-level reliability is not considered. The derived value for $R^{max}(t)$ in Case I is 210 crew/hour. The fourth and fifth columns of Table II respectively show the optimal schedule of resources that need to be allocated to damaged substations in Case I and II; while the fourth and fifth columns of Table III respectively represent allocation of resources to transmission lines in Cases I and II. The adjacent allocation schedules to each component are merged, while the overlapping allocations are removed from the results (associated costs are also deducted in the latter case). As shown in the results, there are components in the system which have multiple allocations, i.e., B87, L91, L100, L131, L132, and L135 in Case I. These multiple allocations perform as a guarantee for the system to different scenarios that can occur when the hurricane strikes. On the other hand, there are components in the system without any allocated resources. The reason is due to low expected cost of damage to these components. This phenomenon also occurs in Case II due to partial restoration which ignores the system-level reliability. Due to economic dynamics of the system, i.e., the cost sensitivity of system to functional state of each component, Case II does not allocate resources to some system components which do not have considerable expected economic risk. Furthermore, presence of redundant components in the system that can compensate the offline state of other components is another reason for observed behavior. However, from the system-level reliability perspective, the restoration scheme of Case II is not always preferable.

For Case III, rather than deriving a scenario-based solution, the expected value of the parameters are plugged into the proposed model. The last columns of Table II and III show the optimal resource allocation in Case III for substations and lines, respectively. As shown in Fig. 1 the total expected cost of restoration in Case III is $15,581,870 which is higher than Cases I and II (i.e., $15,343,980 and $15,065,320, respectively). Therefore, the value of stochastic solution which is the difference between the stochastic solution and the expected value solution is $237,890 and $516,550, for Cases I and II, respectively. The expected load interruptions for Cases I, II, and III are 1,469 MWh, 1,467 MWh, and 1,761 MWh, respectively. While the expected load interruption cost for Case III is significantly higher than Cases I and II, the generation cost of Case I is slightly lower than two other cases.

TABLE I
PROBABILITY OF DAMAGE AND SCALE PARAMETER OF TIME TO REPAIR FOR GENERATING UNITS

Unit Number	Damage Probability	TTR Scale Parameter
$G26$	0.15	8
$G38$	0.35	8
$G39$	0.50	16
$G40$	0.45	16
$G41$	0.35	8
$G42$	0.20	8
$G43$	0.80	12
$G44$	0.25	12
$G45$	0.40	12

As shown in Fig. 2, the optimal resource level for all three cases starts with a high value, but is dramatically dropped by the end of the fist working shift. As results show, Cases I and II have a similar pattern for optimal resource allocation from the beginning of shift 2 until the end of shift 3. The total resource costs for Cases I, II, and III are $373,575, $114,450, and $70,725, respectively. While Case I has the highest, and Case III has the lowest resource allocation cost, Case II has the most cost-effective strategy to restore the system. However, due to contingency of the system to unexpected failures and faults, the partial restoration strategy of Case II does not provide the desired system-level reliability in the normal operating condition. On the other hand, the full restoration strategy of Case I provides higher system-level reliability in expense of a higher resource allocation cost. Considering this trade-off, decision makers can choose the desirable strategy based on their system operation preferences.

TABLE II
PROBABILITY OF DAMAGE, TIME TO REPAIR PARAMETER, AND THE DERIVED RESOURCE ALLOCATION FOR BUSES

Bus Number	Damage Probability	TTR Scale Parameter	Schedule (Case I)	Schedule (Case II)	Schedule (Case III)
$B62$	0.70	10	1-10	1-10	1-7
$B85$	0.20	10	1-9	1-9	1-3
$B86$	0.40	10	1-8	1-8	1-6
$B87$	0.15	7	25-28 46-65	N/A	9-10
$B88$	0.1	10	N/A	N/A	3-4
$B89$	0.05	10	96-107	N/A	1-2
$B90$	0.60	10	1-18	1-18	1-7
$B91$	0.10	7	3-4	5-6	6-7
$B92$	0.30	10	1-4	1-4	1-3
$B93$	0.05	7	1-8	1-8	2-3
$B94$	0.40	10	1-30	1-30	1-5
$B95$	0.20	10	1-8	1-8	2-4
$B96$	0.25	10	1-11	1-11	1-4

TABLE III
PROBABILITY OF DAMAGE, TIME TO REPAIR SCALE PARAMETER, AND THE DERIVED RESOURCE ALLOCATION FOR LINES

Line Number	Damage Probability	TTR Scale Parameter	Schedule (Case I)	Schedule (Case II)	Schedule (Case III)
$L91$	0.20	10	1-3 67-90	N/A	3-5
$L92$	0.30	10	41-63	N/A	5-8
$L100$	0.10	10	1-2 25-38	N/A	1-2
$L101$	0.35	10	1-8	N/A	5-8
$L131$	0.70	10	46-87 1-4	N/A	2-9
$L132$	0.55	10	1-8 72-81	1-8	1-7
$L133$	0.30	15	1-3	2-4	1-6
$L134$	0.25	15	5-7	N/A	2-6
$L135$	0.35	10	1-13 95-107	1-13	3-7
$L136$	0.20	10	2-6	N/A	1-3
$L137$	0.20	15	44-74	N/A	1-4
$L138$	0.15	15	70-89	N/A	4-7
$L139$	0.15	15	24-38	N/A	4-7
$L140$	0.35	10	97-106	N/A	1-5

V. CONCLUSION

A stochastic model to support decision making process for power system restoration in pre-hurricane phase was introduced. The model was formulated as a two-stage stochastic problem with complete recourse. After scenario reduction, the large scale equivalence of the universe problem was solved using Benders' decomposition. Two strategies, i.e. the full restoration, and the partial restoration strategies were analyzed; and the value of stochastic was calculated. The value of stochastic solution as an index, obviously justifies the advantage of obtaining stochastic solution over expected value solution. The numerical results demonstrates the merits and disadvantages of each strategy. While the partial restoration strategy provides the more cost-effective restoration plan, it

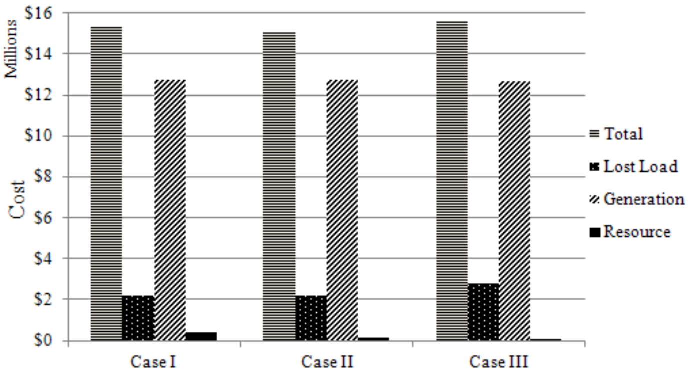

Fig. 1. Expected cost breakdown for three scenarios.

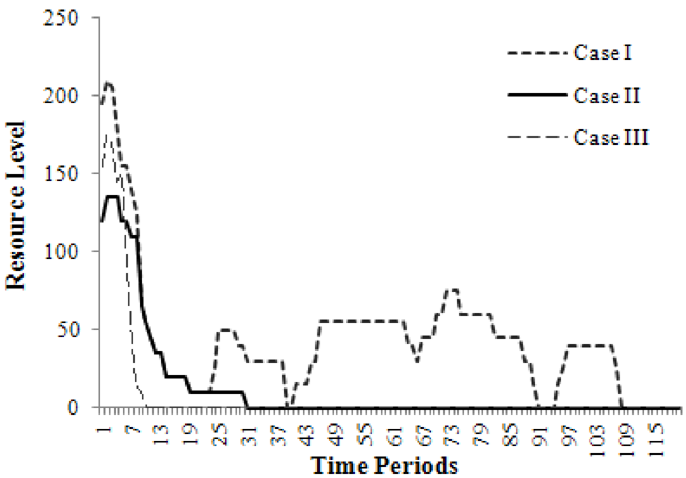

Fig. 2. Optimal resource level for for three scenarios.

may not provide the same system-level reliability that full restoration strategy secures. However, decision makers can choose the best strategy based on operations policy of the utility company.

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